九十五學年第二學期 PHYS2320 電磁學 期末考試題(共二頁)

[Griffiths Chaps. 10 and 12] 2007/06/14, 10:10am–12:00am, 教師：張存績

記得寫上學號，班別及姓名等。請依題號順序每頁答一題。

1. Explain the following terms as clear as possible.
   (a) Einstein's postulates for the special relativity (4%)
   (b) Lorentz gauge and Coulomb gauge (4%)
   (c) Retarded potentials (4%)
   (d) Lienard-Wiechert potentials (4%)
   (e) Invariant quantity and conserved quantity (4%)

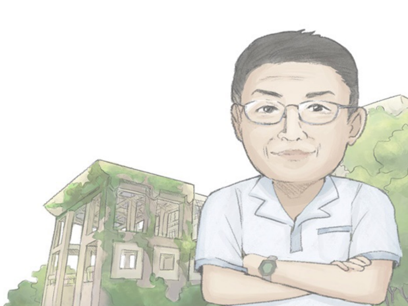
2. (5%, 5%, 5%, 5%) Suppose V=0 and $\mathbf{A}=A_0\cos(kx-\omega t)\,\hat{\mathbf{z}}$, where A_0, ω, and k are constants.
(a) Find **E** and **B**.
(b) What condition must you impose on ω and k?
(c) Use the gauge function $\lambda = xt$ to transform the potentials V' and $\mathbf{A}'$.
(d) Find the new $\mathbf{E}'$ and $\mathbf{B}'$. Comment on the gauge freedom.

3. (10%, 10%) A plane wave is traveling in a media with velocity v_0 in the x-direction. The electric and magnetic fields are given by

$$\mathbf{E}(x,t) = E_0\cos(kx-\omega t)\hat{\mathbf{y}},\quad \mathbf{B}(x,t) = \frac{1}{v_0}E_0\cos(kx-\omega t)\hat{\mathbf{z}}$$

(a) If a particle travels in the x-direction with velocity v_0, what are the new fields $\bar{\mathbf{E}}$ and $\bar{\mathbf{B}}$ that the particle will experience?
(b) Is it possible to find another system in which the magnetic field is zero ($\bar{\mathbf{B}}=0$)?

[Hint: $$\begin{aligned}&\bar{E}_x = E_x,\quad \bar{E}_y = \gamma(E_y - vB_z),\quad \bar{E}_z = \gamma(E_z + vB_y)\\ &\bar{B}_x = B_x,\quad \bar{B}_y = \gamma(B_y + \frac{v}{c^2}E_z),\quad \bar{B}_z = \gamma(B_z - \frac{v}{c^2}E_y)\end{aligned}$$]



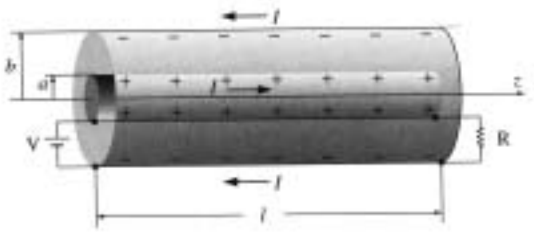
4. (10%, 10%) As the outlaws escape in their getaway car, which goes $\frac{3}{5}c$, the police officer fires a bullet from the pursuit car, which only goes $\frac{1}{2}c$. The muzzle velocity of the bullet (relative to the gun) is $\frac{1}{8}c$. Does the bullet reach its target (a) according to Galileo, (b) according to Einstein?

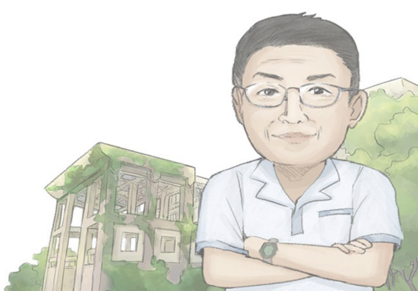
5. (7%, 7%, 6%) A long coaxial cable, of length l, consists of an inner conductor (radius a) and an outer conductor (radius b). It is connected to a battery at one end and a resistor at the other. The inner conductor carries a uniform charge per unit length λ and a steady current I to the right; the outer conductor has the opposite charge and current.

(a) What is the electric field and magnetic field?

(b) Find the total momentum of all charges in the loop. [i.e. hidden momentum]

(c) Find the momentum in the field. Does it account for the hidden momentum?





1.

(a) Einstein's postulates for the special relativity

1. The principle of relativity: All physical laws have the same form in all inertial frames.

2. The universal speed of light: The speed of light in free space is the same in all inertial frames. It does not depend on the motion of the source or the observer.

(b) The Lorentz gauge $\nabla\cdot\mathbf{A}+\mu_0\varepsilon_0\dfrac{\partial V}{\partial t}=0$:

It treat V and $\mathbf{A}$ on an equal footing and is particularly nice in the context of special relativity.

The coulomb gauge $\nabla\cdot\mathbf{A}=0$:

The scalar potential is particularly simple to calculate, but the vector potential is very difficult. It is suitable for the static case.

(c) $V(\mathbf{r},t)=\dfrac{1}{4\pi\varepsilon_0}\displaystyle\int\frac{\rho(\mathbf{r}',t_r)}{R}d\tau'$

$\mathbf{A}(\mathbf{r},t)=\dfrac{\mu_0}{4\pi}\displaystyle\int\frac{\mathbf{J}(\mathbf{r}',t_r)}{R}d\tau', R=|\mathbf{r}-\mathbf{r}'|$

(d) Lienard-Wiechert potentials are the scalar and vector potentials for a moving point charge.

$$V(\mathbf{r},t)=\frac{1}{4\pi\varepsilon_0}\frac{qc}{Rc-\mathbf{R}\cdot\mathbf{v}}$$

$$\mathbf{A}(\mathbf{r},t)=\frac{\mu_0}{4\pi}\frac{qc\mathbf{v}}{Rc-\mathbf{R}\cdot\mathbf{v}}, R=|\mathbf{r}-\mathbf{r}'|$$

(e) Invariant quantity: same value in all inertial frames.

Conserved quantity: same value before and after some process.

2. (a) [Hint: $\mathbf{A}'=\mathbf{A}+\nabla\lambda,\quad V'=V-\partial\lambda/\partial t$]

$$\mathbf{E}=-\nabla V-\frac{\partial\mathbf{A}}{\partial t}=-\omega A_0\sin(kx-\omega t)\hat{z}$$

$$\mathbf{B}=\nabla\times\mathbf{A}=\begin{vmatrix}\hat{x} & y & \hat{z}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ 0 & 0 & A_0\cos(kx-\omega t)\end{vmatrix}=A_0k\sin(kx-\omega t)\hat{y}$$

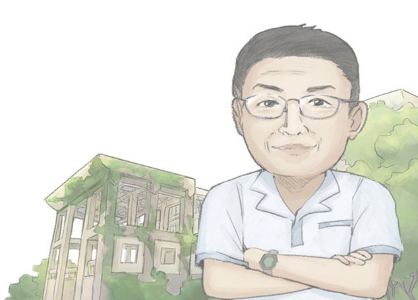
(b)Check:將(a)小題求出的 $\mathbf{E}$ 和 $\mathbf{B}$ 代入 Maxwell equations

$\nabla\cdot\mathbf{E}=0$ ok! $\quad\nabla\cdot\mathbf{B}=0$ ok! $\quad\nabla\times\mathbf{E}=-\dfrac{\partial\mathbf{B}}{\partial t}$ ok!

$\nabla\times\mathbf{B}=\mu_0\varepsilon_0\dfrac{\partial\mathbf{E}}{\partial t}$ ok! , $k^2=\mu_0\varepsilon_0\omega^2$, $\omega=ck$

(c) $\mathbf{A}'=\mathbf{A}+\nabla\lambda=\mathbf{A}+t\hat{\mathbf{x}}=A_0\cos(kx-\omega t)\hat{\mathbf{z}}+t\hat{\mathbf{x}}\qquad V'=V-\dfrac{\partial\lambda}{\partial t}=V-x=-x$

(d) $\mathbf{E}'=-\nabla V-\dfrac{\partial\mathbf{A}'}{\partial t}=\hat{\mathbf{x}}-\omega A_0\sin(kx-\omega t)\hat{\mathbf{z}}+\hat{\mathbf{x}}=-\omega A_0\sin(kx-\omega t)\hat{\mathbf{z}}=\mathbf{E}$



$$\mathbf{B}' = \nabla \times \mathbf{A}' = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ t & 0 & A_0 \cos(kx - \omega t) \end{vmatrix} = A_0 k \sin(kx - \omega t)\hat{\mathbf{y}} = \mathbf{B}$$

3.

(a) $$\begin{cases} \bar{E}_x = 0 \\ \bar{E}_y = \gamma(E_0 \cos(kx - \omega t) - v_0 \frac{1}{v_0} E_0 \cos(kx - \omega t)) = 0, \\ \bar{E}_z = 0 \end{cases}$$

$$\begin{cases} \bar{B}_x = 0 \\ \bar{B}_y = 0 \\ \bar{B}_z = \gamma(\frac{1}{v_0} E_0 \cos(kx - \omega t) - \frac{v_0}{c^2} E_0 \cos(kx - \omega t)) \\ \quad = \frac{1}{\gamma}\frac{1}{v_0} E_0 \cos(kx - \omega t) = \frac{1}{\gamma}\frac{1}{v_0} E_0 \cos(k\gamma(\bar{x} + v_0 \bar{t}) - \omega\gamma(\bar{t} + \frac{v_0 \bar{x}}{c^2})) \\ \quad = \frac{1}{\gamma}\frac{1}{v_0} E_0 \cos(\gamma(k - \frac{\omega v_0}{c^2})\bar{x} - \gamma(\omega - k v_0)\bar{t}) \\ \quad = \frac{1}{\gamma}\frac{1}{v_0} E_0 \cos(\frac{k}{\gamma}\bar{x}), \omega = v_0 k \end{cases}$$

(b) $$\bar{B}_z = 0 \rightarrow \gamma(B_z - \frac{v}{c^2} E_y) = 0 \rightarrow \frac{1}{v} E_0 \cos(kx - \omega t) - \frac{v}{c^2} E_0 \cos(kx - \omega t) = 0$$

$$\rightarrow \frac{1}{v} - \frac{v}{c^2} = 0 \rightarrow v = c$$

但是無法到達光速，所以不可能

4.

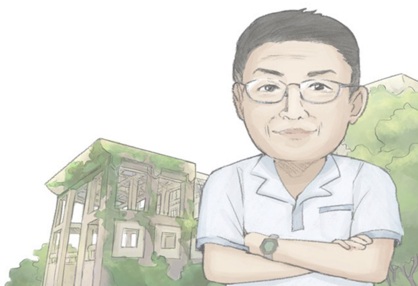
(a) $\frac{1}{2}c + \frac{1}{8}c = \frac{5}{8}c = \frac{25}{40}c \ldots v_b$ 對地

$\frac{3}{5}c = \frac{24}{40}c \ldots v_g$, so $v_b > v_g$ bullet does reach target.

(b) $\dfrac{\frac{1}{2}c + \frac{1}{8}c}{1 + \frac{1}{2} \cdot \frac{1}{8}} = \frac{10}{17}c = \frac{50}{85}c \ldots v_b$ 對地

$\frac{3}{5}c = \frac{51}{85}c \ldots v_g$, so $v_g > v_b$ bullet does NOT reach target.

5.



(a) Gauss's law: $\mathbf{E}=\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{s}\hat{\mathbf{s}}$; Ampere's law: $\mathbf{B}=\frac{\mu_0}{2\pi}\frac{I}{s}\hat{\boldsymbol{\varphi}}$

(b) The current is the same in all segments $I=\lambda u$: $I=\frac{eN_+}{l}u_+=\frac{eN_-}{l}u_-$ so $N_\pm u_\pm=\frac{Il}{e}$.

Relativistic momentum is: $p=\gamma_+ mN_+u_+-\gamma_- mN_-u_-=(\gamma_+-\gamma_-)m\frac{Il}{e}$

The gain in energy (γmc^2) is equal to the work done by the electric force **E**.

$$(\gamma_+-\gamma_-)mc^2=\int_a^b eEdr=\frac{e\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}$$

$$(\gamma_+-\gamma_-)=\frac{e\lambda}{2\pi\varepsilon_0 mc^2}\ln\frac{b}{a}\quad\Rightarrow\quad p=\frac{Il\lambda}{2\pi\varepsilon_0 c^2}\ln\frac{b}{a}$$

or

$$(\gamma_+-\gamma_-)mc^2=\int_a^b eEdr=eV$$

$$(\gamma_+-\gamma_-)=\frac{eV}{mc^2}\quad\Rightarrow\quad p=\frac{IVl}{c^2}$$

$$\mathbf{p}=-\frac{\mu_0 Il\lambda}{2\pi}\ln\frac{b}{a}\hat{\mathbf{z}}$$

(c) The Poynting vector: $\mathbf{S}\equiv\frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B})=\frac{\lambda I}{4\pi^2\varepsilon_0 s^2}\hat{\mathbf{z}}$

The momentum in the fields is: $\mathbf{p}_{\text{em}}=\int\mu_0\varepsilon_0\mathbf{S}d\tau=\frac{\mu_0\lambda I}{4\pi^2}\int_a^b\frac{1}{s^2}l2\pi sds\hat{\mathbf{z}}=\frac{\mu_0\lambda Il}{2\pi}\ln(b/a)\hat{\mathbf{z}}$

The total momentum is zero.